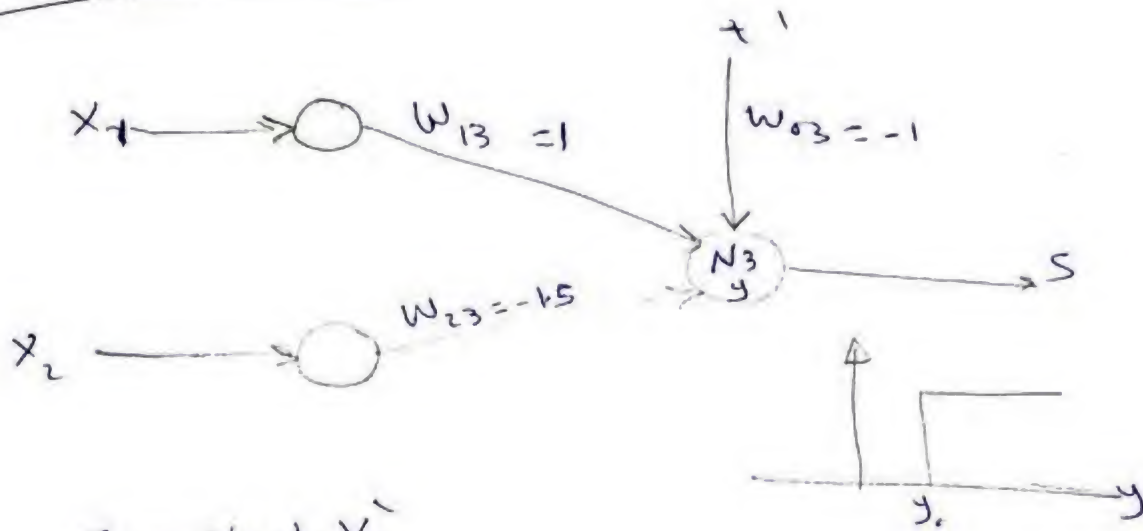


Neural Lec 5

Quiz Sol



$$S = x_1 + x_2$$

$$y = x_1 w_{13} + x_2 w_{23} + w_{03}$$

$$= x_1(1) + x_2(-1.5) - 1$$

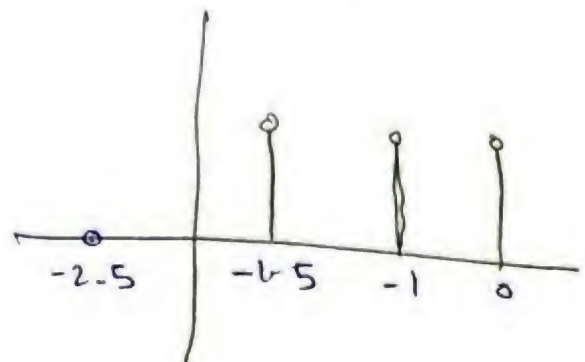
at $x_1 = x_2 = 0$

$$y = -1 \longrightarrow S = 1$$

x_1	x_2	x_2'	S
0	0	1	1
0	1	0	0
1	0	1	1
1	1	0	1

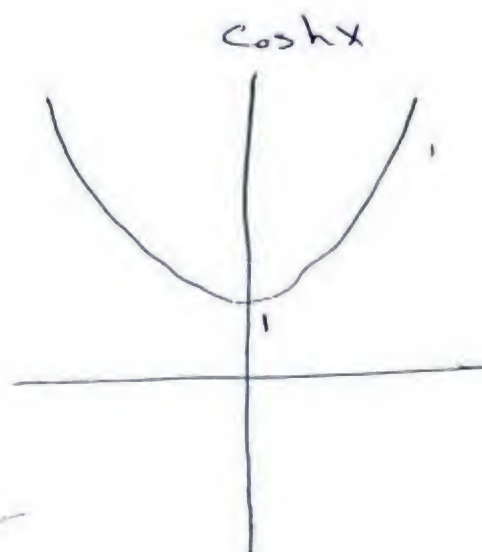
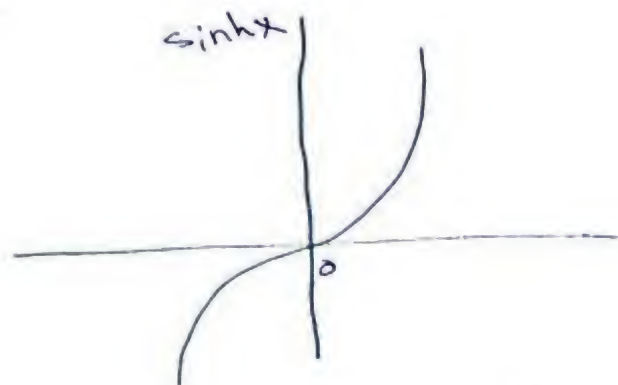
at $x_1 = 1 \quad x_2 = 0$

$$y = -2.5 \longrightarrow S = 0$$



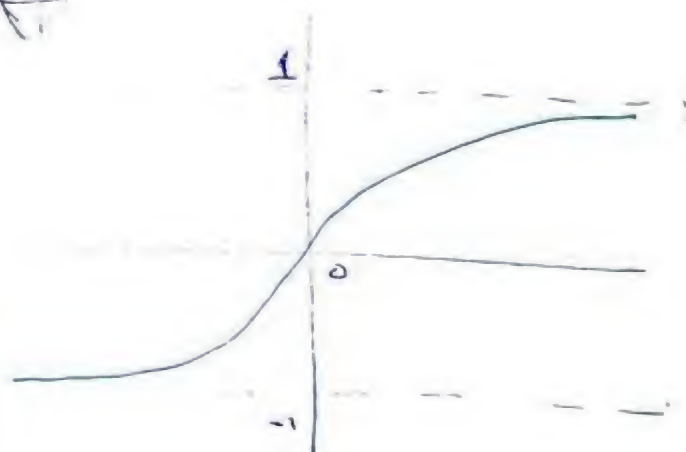
Hyperbolic tangent Function (tanh)

$$\tanh x = \frac{\sinh x}{\cosh x}$$



For a bipolar sigmoidal function with $\alpha=2$

$$\tanh x = \frac{2}{1 + e^{-2x}} - 1$$



→ ~~Hyperbolic~~ Hyperbolic tangent function is the same as a bipolar sigmoidal function

$$g(x) = \frac{2}{1 + e^{-2x}} - 1 \quad (\alpha=2)$$

$$\tanh\left(\frac{\alpha x}{2}\right) = \frac{2}{1 + e^{-2x}} - 1$$

so the bipolar sigmoidal function $g(x) = \frac{2}{1 + e^{-\alpha x}} - 1$
 \equiv hyperbolic tangent function of the form

$$\tanh\left(\frac{\alpha x}{2}\right)$$

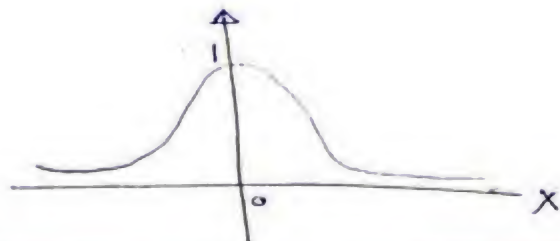
$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x = 1 - \tanh^2 x$$

$$\text{sech}^2 x = 1 - \tanh^2 x \quad \uparrow$$

Let $\tanh x = s$ & $x = y$

$$\frac{d}{dy}(s) = 1 - s^2$$

$$\text{sech } x = \frac{1}{\cosh x}$$



[Ex]

$$x_1 = 0.7$$

$$w_1 = 1.5$$

+1

$$w_0 = 0.8$$

N

s

$$w_2 = 1.5$$

hyperbolic tangent

$$x_2 = 0.9$$

$$s = \tanh y$$

[3]

Activation,

$$y = (0.7)(1.5) + (-1.5)(0.9) + 0.8 = 0.5$$

output signal, for a hyperbolic tangent

$$\text{Function, } s = \tanh y = \tanh 0.5 = 0.462$$

← إذا كانت w_1, w_2 قيم ثابتة عن القيم

المساوية، بينما w_0 (bias weight) w_0 (اعتبارها Param.

للتعويض، أدب قيمة w_0 إذا كانت الإشارة الناتجة 0.81

$$s = \tanh y \Rightarrow y = \tanh^{-1} s$$

$$s = 0.81 \Rightarrow y = 1.127$$

$$y = (0.7)(1.5) + (0.9)(-1.5) + w_0$$

$$\therefore w_0 = 1.427$$

→ Find value of derivative of o/p signal with respect to the activation

$$s = \tanh y$$

$$\frac{ds}{dy} = \frac{d(\tanh y)}{dy}$$

$$\frac{ds}{dy} = \text{sech}^2 y = 1 - \tanh^2 y$$

$$\frac{ds}{dy} = 1 - s^2, \quad s = 0.81$$

$$\boxed{\frac{ds}{dy} = 0.344}$$

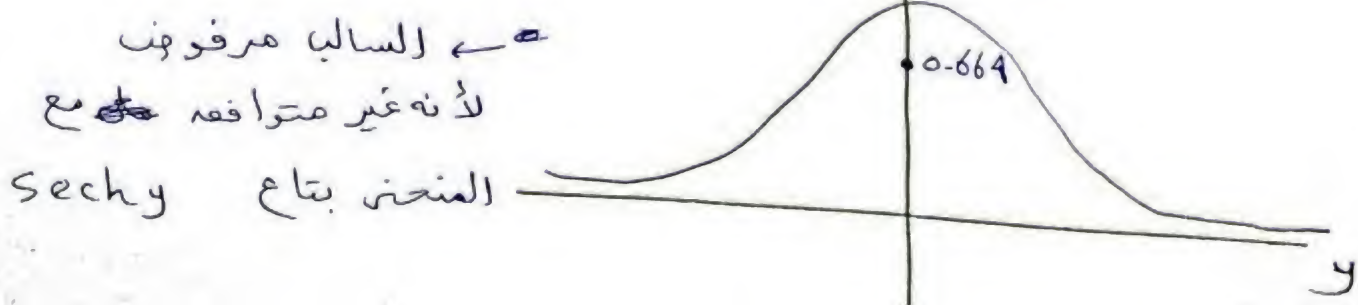
* A neuron employs a hyperbolic tangent function under certain operating condition, the derivative of the old signal s with respect to the activation y is found to be 0.441, find y, s

[sol.]

$$s = \tanh y$$

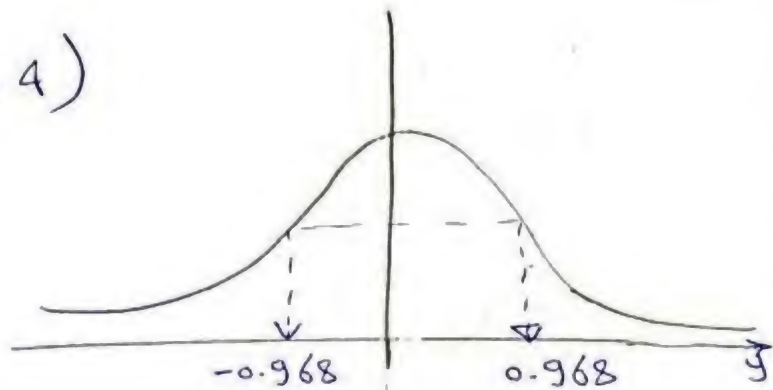
$$\frac{ds}{dy} = \text{sech}^2 y = 0.441$$

$$\text{sech } y = \sqrt{0.441} = \pm 0.664$$



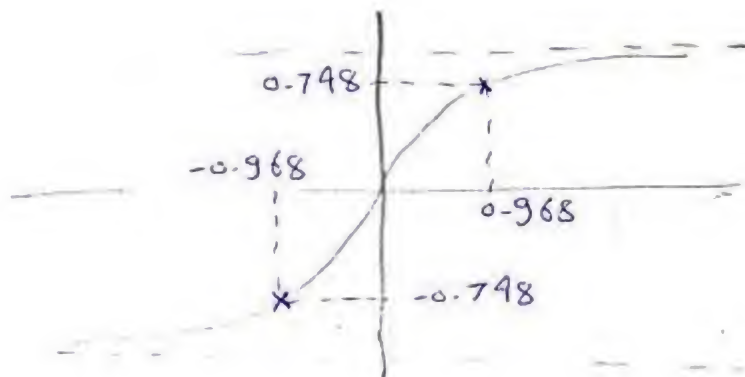
$$y = \operatorname{sech}^{-1}(0.664)$$

$$= \pm 0.968$$



$$s = \tanh y$$

$$s = \pm 0.748$$



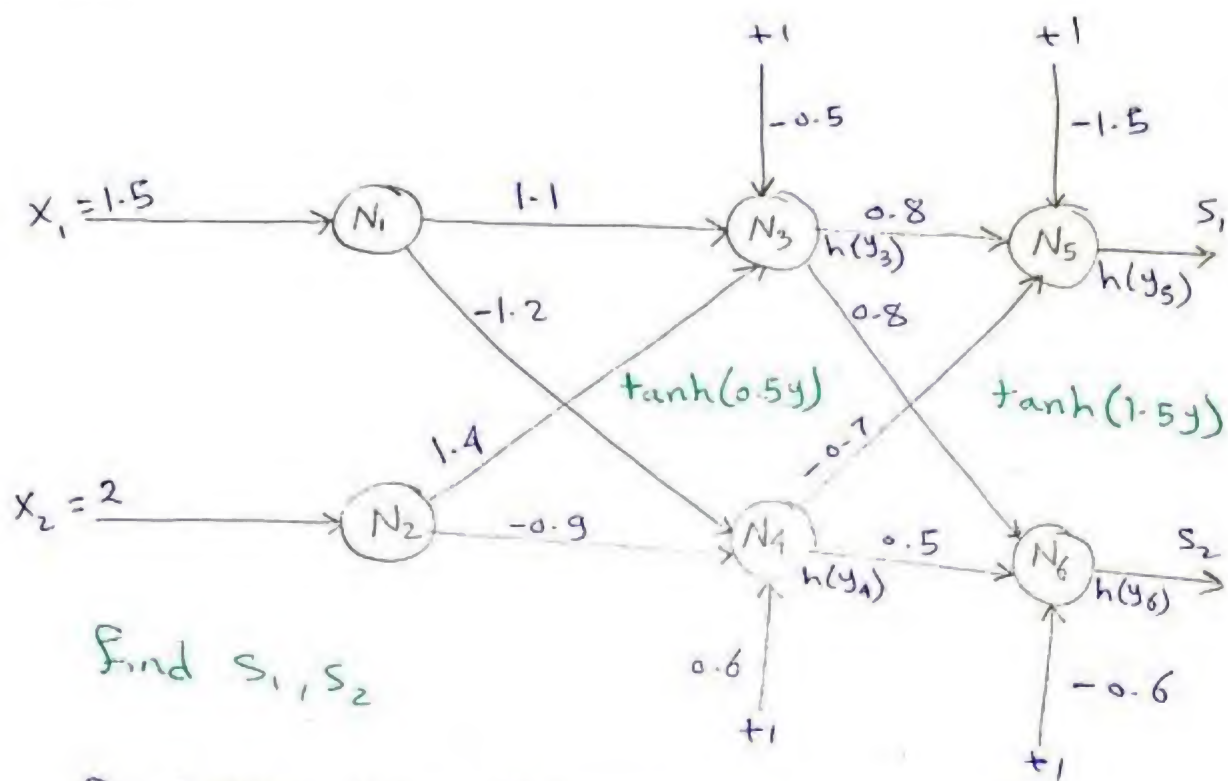
لہٰذا ہم یہ مسئلہ بطریقہ آخری کا حل دیتے ہیں۔

$$s = \tanh y \quad ; \quad \frac{ds}{dy} = 1 - \tanh^2 y = 0.441$$

$$\tanh y = \sqrt{1 - 0.441} = \pm 0.748$$

$$y = \tanh^{-1} s = \tanh^{-1}(\pm 0.748)$$

$$= \pm 0.968$$



Find s_1, s_2

For hidden layer (hyperbolic) $\rightarrow \alpha = 0.5$

For output " (") $\rightarrow \alpha = 1.5$

For hidden neuron N_3 ($\alpha = 0.5$)

Activation,

$$y_3 = (1.5)(1.1) + (2)(1.4) - 0.5 = 3.95$$

output,

$$h(y_3) = \tanh(0.5y_3) = 0.962$$

For hidden neuron N_4 ($\alpha = 0.5$)

Activation,

$$y_4 = 2(-0.9) + (1.5)(-1.2) + 0.6 = -3$$

output,

$$h(y_4) = \tanh(0.5 y_4) = -0.905$$

For output neuron N_5 ($\alpha = 1.5$)

Activation,

$$y_5 = (0.962)(0.8) + (-0.905)(-0.7) - 1.5 = -0.097$$

output,

$$S_1 = h(y_5) = \tanh(1.5 y_5) = \boxed{-0.144}$$

For output neuron N_6 ($\alpha = 1.5$)

Activation,

$$y_6 = (0.962)(0.8) + (-0.905)(0.5) - 0.6 \\ = -0.283$$

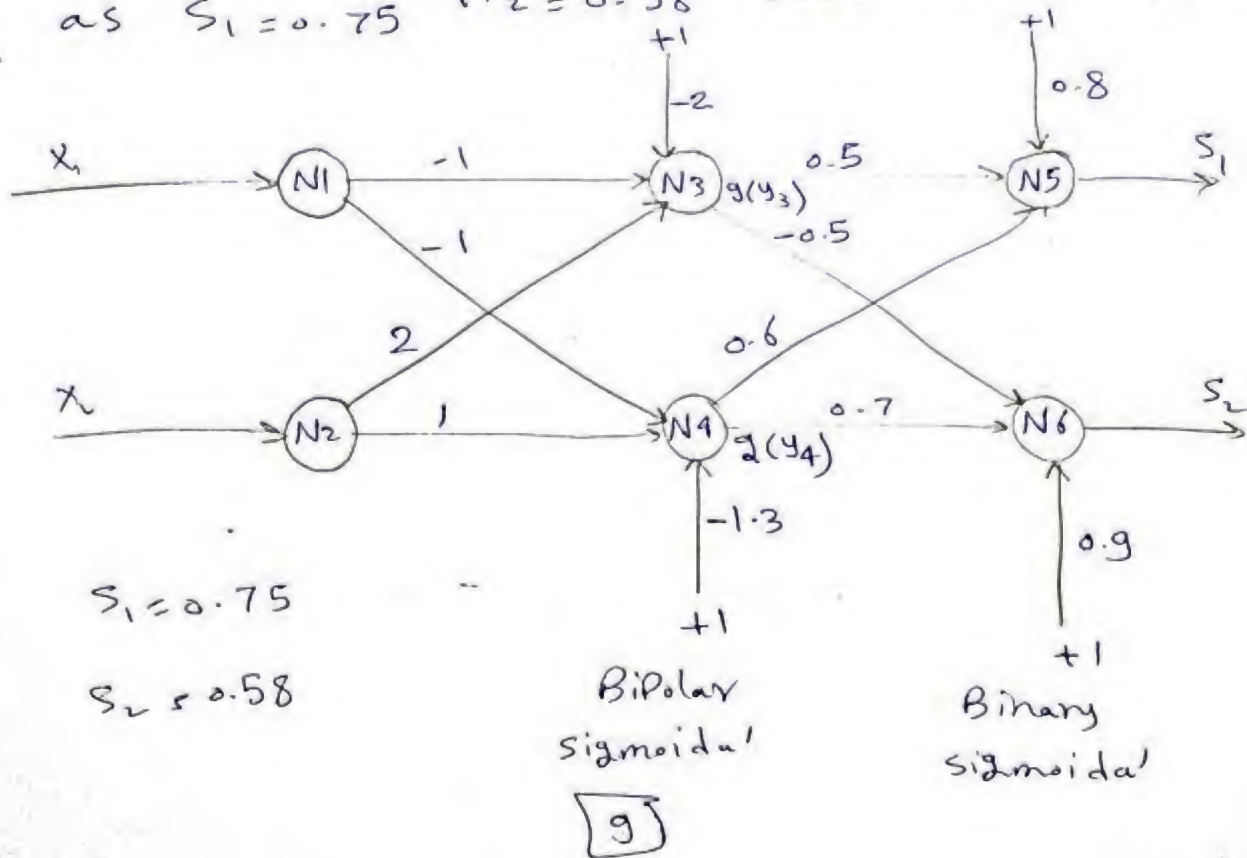
output,

$$h(y_6) = S_2 = \tanh(1.5 y_6) = -0.401$$

لن نستطيع في كثير من الأحوال أن نوجد قيم ال (o/p's) للشبكة لمعرفة ال (i/p's)

→ Given the outputs of a neural network, find the inputs,

→ In the two-input, two output neural network shown, the hidden neurons employ bipolar sigmoidal functions while the output neurons employ binary sigmoidal functions, if the outputs are measured as $S_1 = 0.75$, $S_2 = 0.58$ find inputs X_1, X_2



output layer (N_5, N_6)

Binary sigmoidal

N_5 :

$$y_5 = \ln \frac{s_1}{1-s_1} = 1.099$$

N_6 :

$$y_6 = \ln \left(\frac{s_2}{1-s_2} \right) = 0.323$$

$$y_5 = (0.5) g(y_3) + (0.6) g(y_4) + 0.8 = 1.099$$

$$\boxed{0.5 g(y_3) + 0.6 g(y_4) = 0.299} \rightarrow (1)$$

$$y_6 = (-0.5) g(y_3) + (0.7) g(y_4) + 0.9 = 0.323$$

$$\boxed{-0.5 g(y_3) + 0.7 g(y_4) = -0.577} \rightarrow (2)$$

Solve (1) & (2)

$$g(y_3) = 0.855 \quad (g(y_4) = -0.214)$$

← تنتقل الآن إلى الـ (hidden layer) فابقوا معنا.

$$y_3 = \ln \left[\frac{1 + g(y_3)}{1 - g(y_3)} \right] = -2.549$$

$$y_4 = \text{Ln} \left[\frac{1 + 2(y_4)}{1 - 2(y_4)} \right] = -0.435$$

$$y_3 = -x_1 + 2x_2 - 2 = 2.549$$

$$\boxed{-x_1 + 2x_2 = 0.549} \rightarrow (u)$$

$$y_4 = -x_1 + x_2 - 1.3 = -0.435$$

$$\boxed{-x_1 + x_2 = 0.865} \rightarrow (b)$$

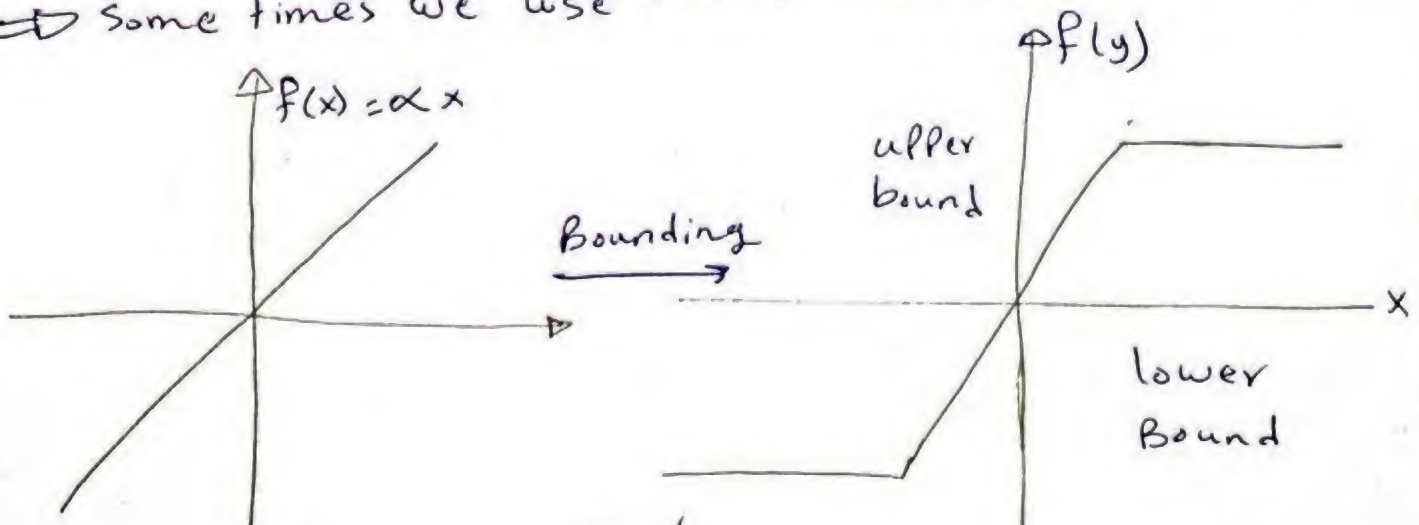
Solve (a) & (b)

$$x_1 = 2.819 \quad \text{and} \quad x_2 = 3.684$$

Summary

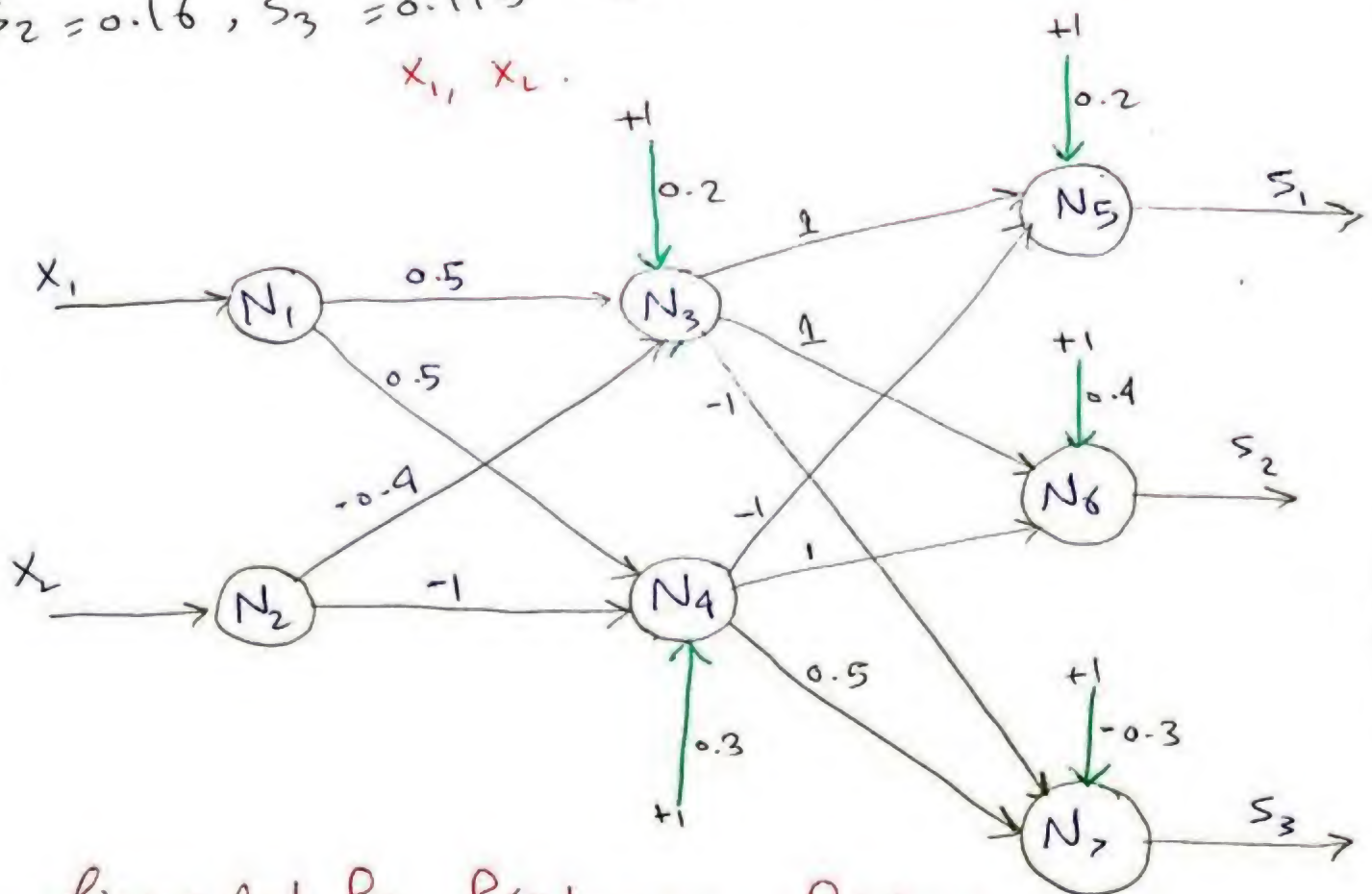
Input \rightarrow output (Forward path) \rightarrow input-hidden-output
 output \rightarrow input (backward path) \rightarrow output-hidden-input

\Rightarrow Some times we use linear Function for activation



EX 7

Consider the two-input, three output neural network shown. The hidden and output neurons employ linear functions of the form $f(x) = \alpha x$, with $\alpha = 0.2$ for each hidden neuron and $\alpha = 1$ for each output neuron. if the outputs are found to be $S_1 = 0.22$, $S_2 = 0.16$, $S_3 = 0.115$ determine the inputs x_1, x_2 .



Linear Act. FN $f(x) = 0.2x$, $f(x) = x$

outputs of the network

$$s_1 = F(y_5) = y_5 = 0.22$$

$$s_2 = F(y_6) = y_6 = -0.16$$

$$s_3 = F(y_7) = y_7 = 0.115$$

Activation of the output neurons,

$$y_5 = (1) F(y_3) + (-1) F(y_4) + 0.2 = 0.22$$

$$\boxed{F(y_3) - F(y_4) = 0.02} \rightarrow (1)$$

$$y_6 = (1) F(y_3) + (1) F(y_4) + 0.4 = -0.16$$

$$\boxed{F(y_3) + F(y_4) = -0.56} \rightarrow (2)$$

$$y_7 = (-1) F(y_3) + (-0.5) F(y_4) - 0.3 = 0.115$$

$$\boxed{F(y_3) + 0.5 F(y_4) = -0.415} \rightarrow (3)$$

لم لدينا 3 معادلات في مجهولين ، وإذا كانت هذه المعادلات

مستقلة عن بعضها البعض فربما لن نتمكن من إيجاد حل .

عدد المعادلات < عدد المجهولين

لهذا ، فإن الشبكة تعمل في ظروف معينة فلا بد أن تكون

هناك في صراعات الشبكة أي تعارض رياضي أو فيزيائي
أو منطقي وبالتالي فإننا ~~لن~~ نتوقع أنه المعادلات
السابقة هي في الواقع معادلتنا (والمعادلة الثالثة معتمدة

عليها) هذا يعني أنه يكفي لإيجاد الجولين

$P(y_3), P(y_4)$ مع معادلتين فقط من المعادلات الثلاثة ~~وذلك~~

وسنجد أنه النتيجة تحققه تلقائياً المعادلة الثالثة.

لـ ~~نحل~~ هنا معادلتين (1, 2)

$$P(y_3) = -0.27 \quad , \quad P(y_4) = -0.29$$

تحققه من أن هاتين القسيتين تحققان المعادلة رقم 3.

Activations of the hidden neurons,

$$y_3 = \frac{P(y_3)}{0.2} = -1.35 = 0.5x_1 - 0.4x_2 - 0.2$$

$$\boxed{0.5x_1 - 0.4x_2 = -1.15} \rightarrow (a)$$

$$y_4 = \frac{P(y_4)}{0.2} = \frac{-0.29}{0.2} = -1.45$$

$$= 0.5x_1 - x_2 + 0.3$$

$$\boxed{0.5x_1 - x_2 = -1.75} \rightarrow (b)$$

$$\text{Solve a \& b} \Rightarrow x_1 = -1.5 \quad , \quad x_2 = 1$$

* A single neuron receives two inputs $X_1 = 0.8$ and $X_2 = 1.2$ with weights $w_1 = 1.6$ & $w_2 = 0.6$ respectively, the bias weight is $w_0 = -1.4$. The neuron employs a hyperbolic tangent function of the form $s = \tanh(\alpha y)$, where s is the output signal, y is the activation, and α is a positive parameter. The derivative of s with respect to y is found to be 0.311. Calculate the values of α , y and s .

Solution

Activation,

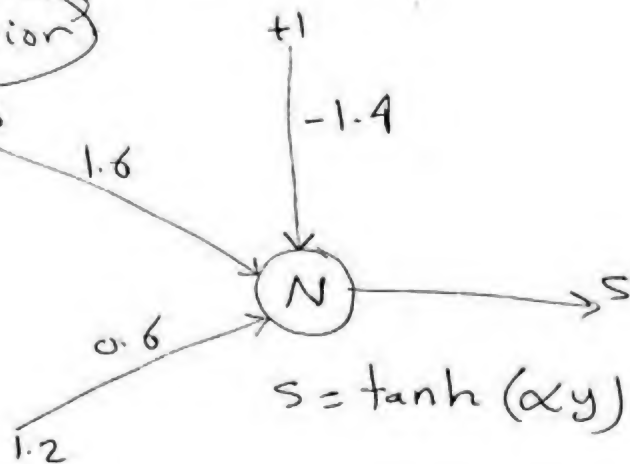
$$y = (0.8)(1.6) + (1.2)(0.6) - 1.4$$

$$= 0.6$$

Derivative,

$$\frac{ds}{dy} = \alpha \operatorname{sech}^2(\alpha y)$$

$$= \alpha [1 - \tanh^2(\alpha y)] = 0.311$$



$$\tanh^2(0.6 \alpha) = 1 - \frac{0.311}{\alpha}$$

يمكننا حل المعادلة السابقة برسم منحني للطرف الأيسر ومثله للطرف الأيمن ونقطة تقاطعها تحدد (α) التي تحدد المعادلة.

α	1	2	3	4	5	6	... ∞
$\tanh^{0.6\alpha} \alpha$	0.288						1
$1 - \frac{0.311}{\alpha}$	0.689						1

$$\alpha = 3$$

output signal

$$s = \tanh(0.6 \times 3)$$

$$= 0.947$$

